JHEP HYPER VERSION
Received: *November 5, 1997*, Accepted: *November 26, 1997*

On central charges and the entropy in matrix theory

Klaus Behrndt
Humboldt University Berlin, Invalidenstrasse 110, 10115 Berlin, Germany
E-mail: behrndt@qft2.physik.hu-berlin.de

Abstract: The Bekenstein-Hawking entropy of BPS black holes can be obtained as the minimum of the mass (= largest central charge). In this letter we investigate the analog procedure for the matrix model of M-theory. Especially we discuss the configurations: (i) $2 \times 2 \times 2$ corresponding to the 5-d black hole and (ii) the $5 \times 5 \times 5$ configuration yielding the 5-d string. After getting their matrix-entropy, we discuss a way of counting of microstates in matrix theory. As Yang Mills field theory we propose the gauged world volume theory of the 11-d KK monopole.

Keywords: Black Holes in String Theory, M(atrix) Theories, Branes in String Theory.

JHEP11(1997)003

Contents

JHEP11(1997)003

1 Introduction

Many aspects of the matrix formulation of M-theory [1] has been discussed in the past year. In this formulation, the compactified M-theory is described by a non-abelian Yang-Mills theory living on the dual torus [2]. Different ways of compactification result in a state degeneracy. For black holes this degeneracy is counted by the Bekenstein-Hawking entropy, given by the area of the horizon. If the BPS bound is saturated, it has been shown that the entropy corresponds to the minimum of the moduli-dependent central charge [3]. Especially, for $N = 2$ black holes [4] this approach has been very fruitful. On the other hand, for Yang-Mills theories it is rather difficult to determine the state degeneracy and it is the aim of this letter to address this question. The central charge of a given Yang-Mills configuration depends on the volumes of wrapped branes, which give the moduli of the configuration. By extremizing this moduli-dependent central charge we find the entropy (state degeneracy). We will especially focus on configurations that correspond to 5-d black holes and the 5-d string. For finite N the 5-d string is compact and their entropy coincides with the Bekenstein-Hawking entropy of the 4-d black hole. In the second part we discuss the microscopic interpretation of the entropy and propose as Yang-Mills theory the worldvolume theory a gauged KK-monopole.

2 Extremal central charges for matrix black holes

The central charge (or the BPS mass) is a functions of the moduli and the minimum gives the entropy. Let us motivate this procedure on the supergravity side and consider the compactification from 11 to 5 dimensions. Wrapping branes around cycles, one can think of the moduli as the asymptotic volumes of these cycles. As we will see below, while extremizing the moduli one has to make sure, that one only varies these cycles and keep all others fixed. Like the internal cycles, also the moduli appear as dual pair, e.g. for a 6-d compact space a wrapped 2-brane around a 2-cycle is dual to a wrapped 5-brane around a 4-cycle and a 2-brane wrapped only over a circle is dual to 5-brane wrapped over a 5-cycle.

To be concrete let us consider the case of the 5-d black hole, which can be obtained by wrapping three M-2-branes. The metric is given by

$$ds^2 = -\frac{1}{(H_1H_2H_3)^{2/3}}dt^2 + (H_1H_2H_3)^{1/3}\left(dr^2 + r^2 d\Omega_3\right) \ . \tag{2.1}$$

with three harmonic functions $H_i = h_i + q_i/r^2$, where q_i are the three electric charges and the $h's$ parameterize the moduli space. We can always choose a coordinate system, which asymptotically becomes the Minkowski space. Thus we can set $h_1h_2h_3 = 1$ and our solution is determined by only two moduli. The mass of this BPS black hole coincides with the susy central charge

$$|Z| = \sum_{i=1}^{3} M_i = (h_1h_2q_3 + h_2h_3q_1 + h_3h_1q_2)/3 = \hat{V}^i q_i. \tag{2.2}$$

where $\hat{V}^i$ are the asymptotic volumes of the 2-cycles. Note, since the $h's$ are dimensionless also the $\hat{V}'s$ have no dimensions, i.e. we have divided out proper powers of the Planck length l_p. This is also convenient, because we want to calculate the entropy (= degeneracy of states) which should be dimensionless. Next, in extremizing this expression we have to vary the 2-volumes (or the dual 4-volumes) and keep the other moduli fix. This means, especially, that we have to fix the total volume of the internal space

$$1 = \hat{V}^6 = \hat{V}^1\hat{V}^2\hat{V}^3 \tag{2.3}$$

($\hat{V}^6$ becomes a modulus if we would wrap a brane around the complete compact space, which is the case for an instantonic wrapped 5-brane).

Now, the entropy can be obtained by extremizing the central charge with respect to $\hat{V}_i$ [3], i.e.:

$$\mathcal{S}_{bh} \sim |Z|^{3/2}_{min} \qquad \text{with} \qquad \left(\frac{\partial}{\partial \hat{V}_i}|Z|\right)_{min} = 0 \ . \tag{2.4}$$

After replacing e.g. of $\hat{V}^3 = 1/(\hat{V}^1\hat{V}^2)$ in (2.2) one finds

$$\mathcal{S}_{bh} \sim \sqrt{q_1q_2q_3} \ , \tag{2.5}$$

which coincides with the Bekenstein-Hawking entropy obtained from the area of the horizon. The analog procedure can also be done for the dual configuration of the magnetic string, which is a bound state of three 5-branes. In this case, because the mass density saturates the BPS bound, one obtains the entropy per unit world volume (i.e. for a fixed point on the world volume).

As next step we are going to discuss the analog procedure for the matrix model. We want to obtain the entropy not by translating of known black hole results, but by extremizing the Yang-Mills central charges. We do not need any information from the black hole or black string – the extremization yields the right result. Again, we want to consider only special examples.

The M-theory, compactified on T^d with the volume V, is described by a Yang-Mills theory living on the dual torus with the volume and Yang Mills coupling constant

$$\Sigma = \frac{l_p^{3d}}{R^d V} = \frac{l_s^{2d}}{V}, \qquad g^2 = \frac{l_p^{3(d-2)}}{R^{d-3} V} = \frac{R^3 \Sigma}{l_p^6}, \tag{2.6}$$

with l_p as Planck length and R is the radius of the $11th$ direction. Again we want to use dimensionless quantities and define

$$\hat{g}^2 = \frac{g^2}{l_s^{d-3}} = \left(\frac{l_s}{l_p}\right)^{d-3} \frac{1}{\hat{V}_d}, \qquad \hat{\Sigma}_d = \frac{\Sigma_d}{l_s^d} = \left(\frac{l_s}{l_p}\right)^{d} \frac{1}{\hat{V}_d}, \tag{2.7}$$

where $\hat{V}_d = V_d / l_p^d$; $l_s^2 = l_p^3/R$. Then the central charges can be written as integrals over the dual Yang-Mills d-torus:

$$\begin{array}{lr}
\hat{Z}^{12} = \frac{1}{\hat{g}^2} \int \hat{\omega} \wedge F = \frac{1}{\hat{g}^2} \hat{\Sigma}^{d-2}\, m_{12} & \text{(transversal 2-brane)} \\
\hat{Z}^{1234} = \frac{1}{\hat{g}^2} \int \hat{\omega} \wedge F \wedge F = \frac{1}{\hat{g}^2} \hat{\Sigma}^{d-4}\, k_{1234} & \text{(wrapped 5-brane)} \\
\hat{Z}^{0} = \frac{1}{\hat{g}^2} \int \hat{\omega} = \frac{\hat{\Sigma}^d}{\hat{g}^2}\, N & \text{(0-branes)} \\
\hat{Z}^{i} = \frac{1}{\hat{g}^2} \int \hat{\omega} \wedge e = \frac{1}{\hat{g}^2} \hat{\Sigma}^{d-1}\, p_i & \text{(longitudinal 2-brane)}
\end{array} \tag{2.8}$$

where we introduced $\int \hat{\omega}_d = \hat{\Sigma}^d$, the flux number m_{12} ($\int_{T^{12}} F = m_{12}$), the instanton number k_{1234} ($\int_{T^{1234}} F \wedge F = k_{1234}$) and the momentum p_i ($\int_{T^i} e = \int F_{0j} F_{ij} + .. = p_i$) (where the integrals include the traces, see also [15]). These central charges coincide with the expressions of the sugra side. Consider, e.g., the transversal 5-brane and using (2.7) we find

$$\hat{Z}^{1234} = \left(\frac{l_s}{l_p}\right) \left(\frac{R}{l_p}\right) \left(\frac{V^{1234}}{l_p^4}\right) k_{1234} = \left(\frac{l_s}{l_p}\right) \hat{R}\, \hat{V}^{1234} k_{1234} \ . \tag{2.9}$$

This is the known central charge (or mass) contribution of a transversal 5-brane, up to the prefactor l_s/l_p which is a consequence of using different parameters to get dimensionless central charges. Introducing again the mass dimensions, both central charges are identical

$$Z_{YM} = \frac{\hat{Z}_{YM}}{l_s} = \frac{\hat{Z}_{grav}}{l_p} = Z_{grav} \ . \tag{2.10}$$

The same procedure can be repeated for the other central charges in (2.8).

Analog to the black holes we will now build bound states and extremize the total central charge. We will start with the analog configuration to the 5-d black hole (2.1), which is a threshold bound state of three 2-branes. Adding up the contributions, the total central charge is given by

$$\hat{Z} = \sum_i \hat{Z}^i = \frac{1}{\hat{g}^2} \int_{V_\Sigma} \omega \wedge F = \frac{1}{\hat{g}^2} (\hat{\Sigma}^1 \hat{\Sigma}^2 m_3 + \hat{\Sigma}^2 \hat{\Sigma}^3 m_1 + \hat{\Sigma}^3 \hat{\Sigma}^1 m_2) \,, \tag{2.11}$$

JHEP11(1997)003

with $i = 1, 2, 3$ counting the different fluxes $(m_1, m_2, m_3) \equiv (m_{12}, m_{34}, m_{56})$ through the different 2-tori Σ^i.

As for the black holes we have to vary only the moduli related to this configuration and keep all others fix, i.e. we vary only 2- or 4-cycle volumes. Especially we have to keep fix the total volume of the Yang-Mills torus and the Yang-Mills coupling constant

$$\hat{\Sigma}^1 \hat{\Sigma}^2 \hat{\Sigma}^3 = \hat{\Sigma}_6 = 1, \qquad \hat{g} = 1. \tag{2.12}$$

Using this relation the minimum of (2.11) is given:

$$\mathcal{S}_{bh}^{YM} \sim |\hat{Z}|_{min}^{3/2} = \sqrt{m_{12} m_{34} m_{56}}\,, \tag{2.13}$$

which coincides with the black hole entropy (2.5) (flux numbers in the matrix model correspond to the membrane charges).

As second example we consider the pure 5-brane configuration $(5 \times 5 \times 5)$, where the 5-branes are wrapped around 4-cycles which pairwise overlap on 2-cycles. This threshold bound state has the total central charge: $((\hat{Z}^1, \hat{Z}^2, \hat{Z}^3) \equiv (\hat{Z}^{1234}, \hat{Z}^{1256}, \hat{Z}^{3456}))$

$$\hat{Z} = \sum_i \hat{Z}^i = \frac{1}{\hat{g}^2} \int_{V_\Sigma} \omega \wedge F \wedge F = \frac{1}{\hat{g}^2} (\hat{\Sigma}^1 k_1 + \hat{\Sigma}^2 k_2 + \hat{\Sigma}^3 k_3). \tag{2.14}$$

Again, in extremizing one has to vary only the 2-cycles, i.e. keeping fix the 6-volume and the coupling constant like in (2.12), so that one obtains

$$\mathcal{S}_{str}^{YM} \sim |\hat{Z}|_{min}^2 = (k_1 k_2 k_3)^{2/3}\ , \tag{2.15}$$

which is the entropy density of the magnetic string. In comparison to the black hole entropy (2.13) the different power here can be understood from the different dimensionality of the horizons, namely the 5-d black hole has an S_3 horizon whereas the string an S_2. Note, for an infinite extended string it does not make sense to talk about the total horizon – for any fixed point of the world volume the horizon is an S_2.

The cases so far correspond to the infinite momentum frame, but what about the finite N case [5]? This means for the string that the radius is finite and effectively describes the 4-d black hole case. In this case we have to add 0-brane contributions to the central charge in (2.14) and get

$$\hat{Z} = \frac{1}{\hat{g}^2} (\hat{\Sigma}^6 N + \hat{\Sigma}^1 k_1 + \hat{\Sigma}^2 k_2 + \hat{\Sigma}^3 k_3)\ . \tag{2.16}$$

This additional contribution also implies that we have to take into account a further modulus. On the sugra side the additional modulus is the radius or better the dimensionless quantity $\hat{R} = R/l_p$. In analogy we can take on the Yang Mills side the dimensionless string length l_s/l_p. But l_s is implicitly contained in all Σ's and in $\hat{g}$, because we used l_s to make these quantities dimensionless, e.g. by $\Sigma^i \to \Sigma^i/l_s^2 = \hat{\Sigma}^i$, see (2.7). Therefore, we should replace l_s, which can be done by $\hat{\Sigma}^i \to l_s^2/l_p^2\,\hat{\Sigma}^i$. Doing this everywhere, also for the gauge coupling, we obtain

$$(l_p/l_s)\,\hat{Z} = \frac{1}{(l_s/l_p)^3 \hat{g}^2} ((l_s/l_p)^6 \Sigma^6 N + (l_s/l_p)^2 (\hat{\Sigma}^1 k_1 + \hat{\Sigma}^2 k_2 + \hat{\Sigma}^3 k_3)) \tag{2.17}$$

(note $\hat{Z}$ has just the inverse prefactor, because it was obtain by $\hat{Z} = l_s Z$) which can be written as

$$\hat{Z} = \frac{1}{\hat{g}^2}\left((l_s/l_p)^4\, \Sigma^6 N + (\hat{\Sigma}^1 k_1 + \hat{\Sigma}^2 k_2 + \hat{\Sigma}^3 k_3)\right) \ . \tag{2.18}$$

The proper constraint is then given by

$$(l_s/l_p)^2\, \hat{\Sigma}^1\hat{\Sigma}^2\hat{\Sigma}^3 = 1, \quad \hat{g} = 1\,, \tag{2.19}$$

which is completely analog to the SUGRA constraint, where the 7-volume has to be fixed:

$$\hat{R}\hat{V}^1\hat{V}^2\hat{V}^3 = (l_p/l_s)^2\hat{V}^1\hat{V}^2\hat{V}^3 = 1\ . \tag{2.20}$$

Note, in terms of (2.7) we could use also the $\hat{V}$'s as moduli. Furthermore, instead of taking l_s as additional moduli one could also take g_0 (the coupling constant of the 0+1 quantum mechanics) and g_s on the SUGRA side. But in any case, one has to fix the gauge coupling $\hat{g}$ resp. the 4-d Newton constant on the SUGRA side.

Using this constraint and extremizing with respect to $\hat{\Sigma}^i$ gives for the entropy

$$\mathcal{S}^{YM}_{4d-bh} \sim |\hat{Z}|^2_{min} = \sqrt{N k_1 k_2 k_3}\,, \tag{2.21}$$

which again coincides entropy of 4-d black holes.

The same procedure can be used also for other configurations. Considering e.g. the configuration of $2\times 5 + mom$ and compactify it on a T^5 we obtain for the central charge:

$$\hat{Z} = \hat{Z}^0 + \hat{Z}^1 + \hat{Z}^{2345} \tag{2.22}$$

and after inserting the expressions and performing the minimization procedure we find:

$$\mathcal{S}^{YM}_{bh} \sim |Z|^{3/2}_{min} = \sqrt{Np\, k_{1234}}\,, \tag{2.23}$$

which is the entropy of the 5-d black hole.

Note that this procedure is not specific to the torus compactification. It should be applicable to much more general cases like $K3$ [6] or even Calabi-Yau compactifications. What one only needs are non-trivial 2- and 4-cycles giving magnetic fluxes and instanton numbers and their radii giving the moduli.

3 The Yang-Mills theory and the microscopic picture

In order to discuss the microscopic interpretation of the entropy (= minimum of the central charge) we have to consider the Yang Mills theory. So far we assumed that a Yang Mills description exists.

As long as one compactifies the M-theory up to a 3-torus, the Yang Mills theory is well defined. However for $d > 3$, one has to address the non-renormalizability of the "standard YM theory". In recent times, one has tried to overcome this problem by considering worldvolume theories of branes, which decouple from the bulk in a certain limit. Hence, they can serve as theories describing compactification for $d > 3$. This has been done for $d \leq 5$ by taking the worldvolume theory of the NS-5-brane [7]. In order to obtain a $d = 6$ compactification one has discussed the worldvolume of the KK monopole in 11-d [8], which has a gauge field enhancement at points where 2-cycles of the Taub-NUT space collapse, see e.g. [9]. Since this theory contains again membranes, it has been suggested to formulate this theory in terms of a matrix model as well [10]. However, in recent discussion [12] it has been argued that for the "standard KK-brane" it is difficult to see how the decoupling can go. The reason is, that in the expected decoupling limit the brane effectively disappears leaving a space with A_{N-1} singularities. This can also be described by interaction with graviton modes (0-branes in 10-d) in the compact KK-direction. Hence, one has to find a way to decouple these modes from the brane world volume.

As discussed in [11] the 11-d worldvolume theory of the KK monopole can be seen as a pure gravitational brane (G-brane) and since the circular isometry has no natural worldvolume interpretation, this is a 6-brane. There is however a subtlety with this brane. Naively one would expect, that a 6-brane in 11-d gives raise to 4 scalars (the 4 transversal directions). However, taking into account also the degrees of freedom of the worldvolume vector, this does not fit in known 7-d supersymmetric theories. Hence, one has to eliminate one degree of freedom. As suggested in [12], this can be done by gauging the circular isometry of the monopole and one obtains as Born-Infeld action

$$S_{KK} \sim \int d^7\xi\, k^2 \sqrt{|\det\left[\partial_i X^\mu \partial_j X^\nu \Pi_{\mu\nu} + k^{-1}(F_{ij} - k^\mu\, \partial_i X^\nu \partial_j X^\rho\, C_{\mu\nu\rho})\right]|} + S_{WZ}, \quad (3.1)$$

where $\Pi_{\mu\nu} = g_{\mu\nu} - k^{-2} k_\mu k_\nu$, $k^2 = k^\mu k^\nu g_{\mu\nu}$ and $g_{\mu\nu}$ is the usual 11-d KK monopole solution ($M_7 \times Taub - NUT$); F_{ij} is the world volume gauge field and k^μ the Killing vector related to the isometry that has been gauged.

After this gauging the coordinate in the isometry direction ($X^\mu = k^\mu$) decouples from the brane, because $\Pi_{\mu\nu}$ is a projector on the space transversal to the Killing vector and the 3-form potential $C_{\mu\nu\rho}$ is contracted with k^μ. Therefore, by gauging, one (isometry) direction has been hidden and one introduced a new parameter k^μ, which scales with the radius of the "hidden direction". Expanding the above action one realizes that k does not enter the gauge field coupling $1/g^2 = M_{Pl}^3$. On the other hand k acts as a coupling constant, like the dilaton in $D = 10$. Thus, taking the limit $k \to \infty$ keeps the worldvolume gauge theory, but suppresses all interactions, especially it decouples the bulk theory. In order to make this statement more explicit, one has to couple the above action to 11-d SUGRA, e.g. by considering the theory $S = S_{11} + S_{KK}$ and investigates the equations of motion (see [12]). In any case sending $k \to \infty$ one has suppressed all gravity. This procedure is very similar to the NS-5-brane worldvolume theory discussed in [7], which is a string theory that decouples from the 10-d bulk theory in the limit of vanishing string coupling and finite string mass. Similar here, the field theory (3.1) decouples from the bulk in the limit of vanishing membrane coupling $1/k$ and finite Planck mass.

For this decoupling it is essential, that one considers the 11-d KK action, i.e. (3.1) contains the 11-d metric and C-field. Compactifying this theory yields in 10-d the known

6-brane solution with k giving the dilaton. But since one has to rescale the metric in going from 11 to 10 dimensions, the 10-d Yang-Mills coupling is dilaton dependent! Note, coming from the 10-d 6-brane one has to decompactify it in a non-trivial way, where the $11th$ direction decouple from the brane (and also corresponding gravitons moving along this direction).

Now we can start with the discussion of the microscopic picture yielding the Yang-Mills entropy discussed in the last section. We will not identify all states neither we discuss the complete U-duality group. Instead, our aim is to give arguments why the entropy formulae are related to the degeneracy of states. For the black holes / black strings this has been done using D-brane techniques, but for the Yang-Mills formulation our understanding is still not yet complete.

The M-theory compactification of $5 \times 5 \times 5$ is described by a bound states of instantons only. For this configuration the Wess-Zumino part, that has to be added to the Born-Infeld action, contains [13]

$$S_{WZ} = \int C_3 \wedge Tr(F \wedge F) \,, \tag{3.2}$$

where C_3 is the 3-form potential. Since a 5-brane corresponds to a non-trivial instanton configuration, it is a source for a (instantonic) membrane on the world volume theory. However this brane is not the usual localized brane, instead it is smeared over a certain region of space time which is mainly given by the instanton size. In the limit of vanishing instanton size the source becomes singular and represents a "standard" membrane lying inside a 6-brane. This is in complete analogy to strings appearing in a 5-brane for vanishing instanton size [14].

It is now tempting to do the microscopic state counting in terms of these brane states, e.g. the instanton number translates into the charge of the membrane or equivalently the number of parallel membranes. Following this procedure, the configuration yielding the entropy $(5 \times 5 \times 5 + mom)$ corresponds in the Yang-Mills picture for vanishing instanton size to a configuration of three membranes intersecting over points. The state degeneracy of the instanton bound state should coincides with the state degeneracy of the membrane bound state, which counts the possibilities of wrapping three membranes around 2-cycles of the 6-torus. In the Yang Mills picture the longitudinal momentum modes counted by N give the rang of the $U(N)$ group. In the analog brane picture for small instantons, it is suggestive to see these modes as 6-branes wrapping the complete T^6. By this procedure, one can reduce the entropy counting to a brane counting using the D-brane technique.

Let us discuss this procedure more explicit for the M-theory configuration $2 \times 5 + mom$ with the $11th$ direction lying along the common worldvolume of the 2- and 5-brane (see also [15], [16]). In the infinite momentum frame this case correspond to the 6-d dyonic string and for finite N the string becomes compact and gives the 5-d black hole. In this case we have to consider a 5+1 dimensional Yang-Mills theory. The translation is now as follows: the (SUGRA) 5-brane corresponds to an instantonic strings and the 2-brane to momentum modes traveling along this string and the momentum is translated to the number of (YM) 5-branes. Therefore, the state counting is reduced to a counting of momentum modes for a string. Again we can argue, that for shrinking instanton size we obtain the known brane configuration of a string lying inside a 5-brane. The statistical entropy that counts string states is $S_{stat.} = 2\pi\sqrt{cp/6}$, where $c = 3/2\, D_{eff}$ and D_{eff} is the effective dimension where one can distribute the momentum modes counted by p. In our case D_{eff} is given by the dimension of the moduli space of k strings (=number of instantons) inside of N 5-branes, which is given by [13] $D_{eff} = 4k(N+1)$. As result the statistical entropy is given by $S_{stat} = 2\pi\sqrt{k(N+1)p}$ and coincides for large N with (2.23).

In conclusion, the aim of this letter was to discuss the state degeneracy (or entropy) in matrix models. Following analog procedures from supersymmetric black holes, we argued that the entropy is given by the minimum of the moduli-dependent central charge. In the second part we discussed a way to count the microstates for the Yang-Mills configuration. The main tool was to employ the fact, that the Yang-Mills fields act as sources for new branes. The degeneracy of these brane configuration can be counted by using the D-brane technique. Note, we were counting Yang-Mills states and not the states of the supergravity side! For this counting we only used the facts: (i) that the Yang-Mills configuration is equivalent to intersecting brane configuration for vanishing instanton size and (ii) that the degeneracy of states should not alter if we shrink the instanton size. As 7-d Yang-Mills theory that is needed for the T^6 compactification, we discussed the world volume theory of a gauged 11-d KK-monopole. The gauging effectively introduces a membrane coupling constant and in the limit of vanishing coupling the 6+1 dimensional field theory decouples from the bulk.

Acknowledgments

I would like to thank especially Joachim Rahmfeld for numerous discussions and for collaborating in early stages of this work. In addition I am grateful Eric Bergshoeff for valuable discussions and the Stanford University for its hospitality, where this work has been done. The work is supported by the Deutsche Forschungsgemeinschaft (DFG).

References

[1] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M theory as a matrix model*, *Phys. Rev.* **D 55** (1997) 5112-5128 [`hep-th/9610043`].

[2] T. Banks, N. Seiberg and S. Shenker, *Branes from Matrices*, *Nucl. Phys.* **B 490** (1997) 91-106 [`hep-th/9612157`].

[3] S. Ferrara and R. Kallosh, *Supersymmetry and attractor*, *Phys. Rev.* **D 54** (1996) 1514-1524 [`hep-th/9602136`]; *Universality of supersymmetric attractors*, *Phys. Rev.* **D 54** (1996) 1525-1534 [`hep-th/9603090`].

[4] K. Behrndt, R. Kallosh, J. Rahmfeld, M. Shmakova and W.K. Wong, *STU black holes and string triality*, *Phys. Rev.* **D 54** (1996) 6293-6301 [hep-th/9608059]; K. Behrndt, G.L. Cardoso, B. de Wit, R. Kallosh, D. Lüst and T. Mohaupt, *Classical and quantum $N = 2$ supersymmetric black holes*, *Nucl. Phys.* **B 488** (1997) 236-260 [hep-th/9610105].

[5] L. Susskind, *Another conjecture about M(atrix) theory*, hep-th/9704080.

[6] W. Fischler and A. Rajaraman, *M(atrix) string theory on K3*, hep-th/9704123.

[7] N. Seiberg, *New theories in 6 dimensions and Matrix description of M-theory in T^5 and T^5/Z_2*, *Phys. Lett.* **B 408** (1997) 98-104 [hep-th/9705221].

[8] I. Brunner and A. Karch, *Matrix Description of M-theory on T^6*, hep-th/9707256.

[9] A. Sen, *A note on enhanced gauge symmetries in M- and string theory*, hep-th/9707123.

[10] A. Losev, G. Moore and S.L. Shatashvili, *M & m's*, hep-th/9707250; A. Hanany and G. Lifschytz, *M(atrix) theory on T^6 and a m(atrix) theory description of KK monopoles*, hep-th/9708037.

[11] C.M. Hull, *Gravitational duality, branes and charges*, hep-th/9705162.

[12] E. Bergshoeff, B. Janssen and T. Ortin, *Kaluza-Klein monopoles and gauged sigma-models*, hep-th/9706117.

[13] M.R. Douglas, *Branes within branes*, hep-th/9512077.

[14] E. Witten, *Small instantons in string theory* , *Nucl. Phys.* **B 460** (1996) 541-559 [hep-th/9511030].

[15] R. Dijkgraaf, E. Verlinde and H. Verlinde, *5D black holes and matrix strings*, hep-th/9704018.

[16] E. Halyo, *M(atrix) black holes in five dimensions*, hep-th/9705107.

[17] A. Sen, *D0-branes on T^n and matrix theory*, hep-th/9709220; N. Seiberg, *Why is the matrix model correct?*, *Phys. Rev. Lett.* **79** (1997) 3577-3580 [hep-th/9710009].